

7-2 Study Guide and Intervention

Division Properties of Exponents

Divide Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a , $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b , $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 Simplify $\frac{a^4b^7}{ab^2}$. Assume that no denominator equals zero.

$$\begin{aligned} \frac{a^4b^7}{ab^2} &= \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right) && \text{Group powers with the same base.} \\ &= (a^{4-1})(b^{7-2}) && \text{Quotient of Powers} \\ &= a^3b^5 && \text{Simplify.} \end{aligned}$$

The quotient is a^3b^5 .

Example 2 Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that no denominator equals zero.

$$\begin{aligned} \left(\frac{2a^3b^5}{3b^2}\right)^3 &= \frac{(2a^3b^5)^3}{(3b^2)^3} && \text{Power of a Quotient} \\ &= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3} && \text{Power of a Product} \\ &= \frac{8a^9b^{15}}{27b^6} && \text{Power of a Power} \\ &= \frac{8a^9b^9}{27} && \text{Quotient of Powers} \end{aligned}$$

The quotient is $\frac{8a^9b^9}{27}$.

Exercises

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{5^5}{5^2} 5^3$ or 125

2. $\frac{m^6}{m^4} m^2$

3. $\frac{p^5n^4}{p^2n} p^3n^3$

4. $\frac{a^2}{a} a$

5. $\frac{x^5y^3}{x^5y^2} y$

6. $\frac{-2y^7}{14y^5} - \frac{1}{7}y^2$

7. $\frac{xy^6}{y^4x} y^2$

8. $\left(\frac{2a^2b}{a}\right)^3 8a^3b^3$

9. $\left(\frac{4p^4r^4}{3p^2r^2}\right)^3 \frac{64}{27}p^6r^6$

10. $\left(\frac{2r^5w^3}{r^4w^3}\right)^4 16r^4$

11. $\left(\frac{3r^6n^3}{2r^5n}\right)^4 \frac{81}{16}r^4n^8$

12. $\frac{r^7n^7t^2}{n^3r^3t^2} r^4n^4$

7-2 Study Guide and Intervention *(continued)*

Division Properties of Exponents

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a , $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n , $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that no denominator equals zero.

$$\begin{aligned} \frac{4a^{-3}b^6}{16a^2b^6c^{-5}} &= \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right) && \text{Group powers with the same base.} \\ &= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5) && \text{Quotient of Powers and Negative Exponent Properties} \\ &= \frac{1}{4}a^{-5}b^0c^5 && \text{Simplify.} \\ &= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5 && \text{Negative Exponent and Zero Exponent Properties} \\ &= \frac{c^5}{4a^5} && \text{Simplify.} \end{aligned}$$

The solution is $\frac{c^5}{4a^5}$.

Exercises

Simplify each expression. Assume that no denominator equals zero.

1. $\frac{2^2}{2^{-3}} \cdot 2^5$ or 32

2. $\frac{m}{m^{-4}} \cdot m^5$

3. $\frac{p^{-8}}{p^3} \cdot \frac{1}{p^{11}}$

4. $\frac{b^{-4}}{b^{-5}} \cdot b$

5. $\frac{(-x^{-1}y)^0}{4w^{-1}y^2} \cdot \frac{w}{4y^2}$

6. $\frac{(a^2b^3)^2}{(ab)^{-2}} \cdot a^6b^8$

7. $\frac{x^4y^0}{x^{-2}} \cdot x^6$

8. $\frac{(6a^{-1}b)^2}{(b^2)^4} \cdot \frac{36}{a^2b^6}$

9. $\frac{(3rt)^2u^{-4}}{r^{-1}t^2u^7} \cdot \frac{9r^3}{u^{11}}$

10. $\frac{m^{-3}t^{-5}}{(m^2t^3)^{-1}} \cdot \frac{1}{mt^2}$

11. $\left(\frac{4m^2n^2}{8m^{-1}l}\right)^0 \cdot 1$

12. $\frac{(-2mn^2)^{-3}}{4m^{-6}n^4} - \frac{m^3}{32n^{10}}$