

7-6 Study Guide and Intervention**Growth and Decay**

Exponential Growth Population increases and growth of monetary investments are examples of **exponential growth**. This means that an initial amount increases at a steady rate over time.

Exponential Growth	<p>The general equation for exponential growth is $y = a(1 + r)^t$.</p> <ul style="list-style-type: none"> • y represents the final amount. • a represents the initial amount. • r represents the rate of change expressed as a decimal. • t represents time.
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Example 1 **POPULATION** The population of Johnson City in 2005 was 25,000. Since then, the population has grown at an average rate of 3.2% each year.

- a. Write an equation to represent the population of Johnson City since 2005.

The rate 3.2% can be written as 0.032.

$$y = a(1 + r)^t$$

$$y = 25,000(1 + 0.032)^t$$

$$y = 25,000(1.032)^t$$

- b. According to the equation, what will the population of Johnson City be in 2015?

In 2015 t will equal 2015 – 2005 or 10.

Substitute 10 for t in the equation from part a.

$$y = 25,000(1.032)^{10} \quad t = 10$$

$$\approx 34,256$$

In 2015 the population of Johnson City will be about 34,256.

Example 2 **INVESTMENT** The Garcias have \$12,000 in a savings account. The bank pays 3.5% interest on savings accounts, compounded monthly. Find the balance in 3 years.

The rate 3.5% can be written as 0.035.

The special equation for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A represents the balance, P is the initial amount, r represents the annual rate expressed as a decimal, n represents the number of times the interest is compounded each year, and t represents the number of years the money is invested.

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{nt} \\ &= 12,000\left(1 + \frac{0.035}{12}\right)^{3(12)} \\ &\approx 13,326.49 \end{aligned}$$

In three years, the balance of the account will be \$13,326.49.

Exercises

- POPULATION** The population of the United States has been increasing at an average annual rate of 0.91%. If the population was about 303,146,000 in 2008, predict the population in 2012.
about 314,332,051
- INVESTMENT** Determine the value of an investment of \$2500 if it is invested at an interest rate of 5.25% compounded monthly for 4 years.
\$3082.78
- POPULATION** It is estimated that the population of the world is increasing at an average annual rate of 1.3%. If the 2008 population was about 6,641,000,000, predict the 2015 population.
about 7,269,417,259
- INVESTMENT** Determine the value of an investment of \$100,000 if it is invested at an interest rate of 5.2% compounded quarterly for 12 years.
\$185,888.87

7-6 Study Guide and Intervention *(continued)*

Growth and Decay

Exponential Decay Radioactive decay and depreciation are examples of **exponential decay**. This means that an initial amount decreases at a steady rate over a period of time.

Exponential Decay	<p>The general equation for exponential decay is $y = a(1 - r)^t$.</p> <ul style="list-style-type: none"> • y represents the final amount. • a represents the initial amount. • r represents the rate of decay expressed as a decimal. • t represents time.
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Example

DEPRECIATION The original price of a tractor was \$45,000. The value of the tractor decreases at a steady rate of 12% per year.

a. Write an equation to represent the value of the tractor since it was purchased.

The rate 12% can be written as 0.12.

$$y = a(1 - r)^t \quad \text{General equation for exponential decay}$$

$$y = 45,000(1 - 0.12)^t \quad a = 45,000 \text{ and } r = 0.12$$

$$y = 45,000(0.88)^t \quad \text{Simplify.}$$

b. What is the value of the tractor in 5 years?

$$y = 45,000(0.88)^t \quad \text{Equation for decay from part a}$$

$$y = 45,000(0.88)^5 \quad t = 5$$

$$y \approx 23,747.94 \quad \text{Use a calculator.}$$

In 5 years, the tractor will be worth about \$23,747.94.

Exercises

- POPULATION** The population of Bulgaria has been decreasing at an annual rate of 0.89%. If the population of Bulgaria was about 7,450,349 in the year 2005, predict its population in the year 2015. **about 6,813,204**
- DEPRECIATION** Mr. Gossell is a machinist. He bought some new machinery for about \$125,000. He wants to calculate the value of the machinery over the next 10 years for tax purposes. If the machinery depreciates at the rate of 15% per year, what is the value of the machinery (to the nearest \$100) at the end of 10 years? **about \$24,600**
- ARCHAEOLOGY** The *half-life* of a radioactive element is defined as the time that it takes for one-half a quantity of the element to decay. Radioactive carbon-14 is found in all living organisms and has a half-life of 5730 years. Consider a living organism with an original concentration of carbon-14 of 100 grams.
 - If the organism lived 5730 years ago, what is the concentration of carbon-14 today? **50 g**
 - If the organism lived 11,460 years ago, determine the concentration of carbon-14 today. **25 g**
- DEPRECIATION** A new car costs \$32,000. It is expected to depreciate 12% each year for 4 years and then depreciate 8% each year thereafter. Find the value of the car in 6 years. **about \$16,242.63**