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7-7 Study Guide and Intervention

Geometric Sequences as Exponential Functions

Recognize Geometric Sequences A geometric sequence is a sequence in which each term after the first is found by multiplying the previous term by a nonzero constant r called the **common ratio**. The common ratio can be found by dividing any term by its previous term.

Example 1 **Determine whether the sequence is *arithmetic, geometric,* or *neither:* 21, 63, 189, 567, . . .**

Find the ratios of the consecutive terms. If the ratios are constant, the sequence is geometric.

$$21 \quad 63 \quad 189 \quad 567$$

$$\frac{63}{21} = \frac{189}{63} = \frac{567}{189} = 3$$

Because the ratios are constant, the sequence is geometric. The common ratio is 3.

Example 2 **Find the next three terms in this geometric sequence: −1215, 405, −135, 45, . . .**

Step 1 Find the common ratio.

$$-1215 \quad 405 \quad -135 \quad 45$$

$$\frac{405}{-1215} = \frac{-135}{405} = \frac{45}{-135} = \frac{-1}{3}$$

The value of r is $-\frac{1}{3}$.

Step 2 Multiply each term by the common ratio to find the next three terms.

$$45 \quad -15 \quad 5 \quad -\frac{5}{3}$$

$$\times\left(-\frac{1}{3}\right) \quad \times\left(-\frac{1}{3}\right) \quad \times\left(-\frac{1}{3}\right)$$

The next three terms of the sequence are –15, 5, and $-\frac{5}{3}$.

Exercises

Determine whether each sequence is *arithmetic, geometric,* or *neither.* Explain.

1. 1, 2, 4, 8, . . .

Geometric; common ratio is 2.

2. 9, 14, 6, 11, . . .

Neither; there is no common difference or ratio.

3. $\frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \ldots$

Geometric; common ratio is $\frac{1}{2}$.

4. –2, 5, 12, 19, . . .

Arithmetic; common difference is 7.

Find the next three terms in each geometric sequence.

5. 648, –216, 72, . . .

$-24, 8, -2\frac{2}{3}$

6. 25, –5, 1, . . .

$-\frac{1}{5}, \frac{1}{25}, -\frac{1}{125}$

7. $\frac{1}{16}, \frac{1}{2}$, 4, . . .

32, 256, 2048

8. 72, 36, 18, . . .

9, $4\frac{1}{2}$, $2\frac{1}{4}$

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7-7 Study Guide and Intervention *(continued)*

Geometric Sequences as Exponential Functions

Geometric Sequences and Functions The nth term a_n of a geometric sequence with first term a_1 and common ratio r is given by the following formula, where n is any positive integer: $a_n = a_1 \cdot r^{n-1}$.

Example **a. Write an equation for the *n*th term of the geometric sequence 5, 20, 80, 320, . . .**

The first term of the sequence is 320. So, $a_1 = 320$. Now find the common ratio.

5 → 20 → 80 → 320

$$\frac{20}{5} = \frac{80}{20} = \frac{320}{80} = 4$$

The common ratio is 4. So, $r = 4$.

$a_n = a_1 \cdot r^{n-1}$	Formula for nth term
$a_n = 5 \cdot 4^{n-1}$	$a_1 = 5$ and $r = 4$

b. Find the seventh term of this sequence.

Because we are looking for the seventh term, $n = 7$.

$a_n = a_1 \cdot r^{n-1}$	Formula for nth term
$a_7 = 5 \cdot 4^{7-1}$	$n = 7$
$= 5 \cdot 4^6$	Simplify.
$= 5 \cdot 4096$	$4^6 = 4096$
$= 20{,}480$	Multiply.

The seventh term of the sequence is 20,480.

Lesson 7-7

Exercises

1. Write an equation for the nth term of the geometric sequence –2, 10, –50, ... Find the eleventh term of this sequence. $a_n = -2 \cdot (-5)^{n-1}$; $-19{,}531{,}250$

2. Write an equation for the nth term of the geometric sequence 512, 128, 32, ... Find the sixth term of this sequence. $a_n = 512 \cdot \left(\frac{1}{4}\right)^{n-1}$; $\frac{1}{2}$

3. Write an equation for the nth term of the geometric sequence $\frac{4}{9}$, 4, 36, ... Find the eighth term of this sequence. $a_n = \frac{4}{9} \cdot 9^{n-1}$; $2{,}125{,}764$

4. Write an equation for the nth term of the geometric sequence 6, –54, 486, ... Find the ninth term of this sequence. $a_n = 6 \cdot (-9)^{n-1}$; $258{,}280{,}326$

5. Write an equation for the nth term of the geometric sequence 100, 80, 64, ... Find the seventh term of this sequence. $a_n = 100 \cdot \left(\frac{4}{5}\right)^{n-1}$; $26\frac{134}{625}$

6. Write an equation for the nth term of the geometric sequence $\frac{2}{5}, \frac{1}{10}, \frac{1}{40}, \ldots$. Find the sixth term of this sequence. $a_n = \frac{2}{5} \cdot \left(\frac{1}{4}\right)^{n-1}$; $\frac{1}{2560}$

7. Write an equation for the nth term of the geometric sequence $\frac{3}{8}, -\frac{3}{2}$, 6, ... Find the tenth term of this sequence. $a_n = \frac{3}{8} \cdot (-4)^{n-1}$; $-98{,}304$

8. Write an equation for the nth term of the geometric sequence –3, –21, –147, ... Find the fifth term of this sequence. $a_n = -3 \cdot 7^{n-1}$; -7203