NAME ______________________ DATE ____________ PERIOD ____________

7-8 Study Guide and Intervention

Recursive Formulas

Using Recursive Formulas A **recursive formula** allows you to find the nth term of a sequence by performing operations on one or more of the terms that precede it.

Example **Find the first five terms of the sequence in which $a_1 = 5$ and $a_n = -2a_{n-1} + 14$, if $n \geq 2$.**

The given first term is $a_1 = 5$. Use this term and the recursive formula to find the next four terms.

$$\begin{aligned} a_2 &= -2a_{2-1} + 14 && n = 2 \\ &= -2a_1 + 14 && \text{Simplify.} \\ &= -2(5) + 14 \text{ or } 4 && a_1 = 5 \end{aligned}$$

$$\begin{aligned} a_3 &= -2a_{3-1} + 14 && n = 3 \\ &= -2a_2 + 14 && \text{Simplify.} \\ &= -2(4) + 14 \text{ or } 6 && a_2 = 4 \end{aligned}$$

$$\begin{aligned} a_4 &= -2a_{4-1} + 14 && n = 4 \\ &= -2a_3 + 14 && \text{Simplify.} \\ &= -2(6) + 14 \text{ or } 2 && a_3 = 6 \end{aligned}$$

$$\begin{aligned} a_5 &= -2a_{5-1} + 14 && n = 5 \\ &= -2a_4 + 14 && \text{Simplify.} \\ &= -2(2) + 14 \text{ or } 10 && a_4 = 2 \end{aligned}$$

The first five terms are 5, 4, 6, 2, and 10.

Exercises

Find the first five terms of each sequence.

1. $a_1 = -4$, $a_n = 3a_{n-1}$, $n \geq 2$

–4, –12, –36, –108, –324

2. $a_1 = 5$, $a_n = 2a_{n-1}$, $n \geq 2$

5, 10, 20, 40, 80

3. $a_1 = 8$, $a_n = a_{n-1} - 6$, $n \geq 2$

8, 2, –4, –10, –16

4. $a_1 = -32$, $a_n = a_{n-1} + 13$, $n \geq 2$

–32, –19, –6, 7, 20

5. $a_1 = 6$, $a_n = -3a_{n-1} + 20$, $n \geq 2$

6, 2, 14, –22, 86

6. $a_1 = -9$, $a_n = 2a_{n-1} + 11$, $n \geq 2$

–9, –7, –3, 5, 21

7. $a_1 = 12$, $a_n = 2a_{n-1} - 10$, $n \geq 2$

12, 14, 18, 26, 42

8. $a_1 = -1$, $a_n = 4a_{n-1} + 3$, $n \geq 2$

–1, –1, –1, –1, –1

9. $a_1 = 64$, $a_n = 0.5a_{n-1} + 8$, $n \geq 2$

64, 40, 28, 22, 19

10. $a_1 = 8$, $a_n = 1.5a_{n-1}$, $n \geq 2$

8, 12, 18, 27, 40.5

11. $a_1 = 400$, $a_n = \frac{1}{2}a_{n-1}$, $n \geq 2$

400, 200, 100, 50, 25

12. $a_1 = \frac{1}{4}$, $a_n = a_{n-1} + \frac{3}{4}$, $n \geq 2$

$\mathbf{\frac{1}{4}, 1, \frac{7}{4}, \frac{5}{2}, \frac{13}{4}}$

7-8 Study Guide and Intervention *(continued)*

Recursive Formulas

Writing Recursive Formulas Complete the following steps to write a recursive formula for an arithmetic or geometric sequence.

Step 1	Determine if the sequence is arithmetic or geometric by finding a common difference or a common ratio.
Step 2	Write a recursive formula. **Arithmetic Sequences** $a_n = a_{n-1} + d$, where d is the common difference **Geometric Sequences** $a_n = r \cdot a_{n-1}$, where r is the common ratio
Step 3	State the first term and the domain for n.

Example **Write a recursive formula for 216, 36, 6, 1, … .**

Step 1 First subtract each term from the term that follows it.

$216 - 36 = 180$ $\quad$ $36 - 6 = 30$ $\quad$ $6 - 1 = 5$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$\frac{36}{216} = \frac{1}{6}$ $\quad$ $\frac{6}{36} = \frac{1}{6}$ $\quad$ $\frac{1}{6} = \frac{1}{6}$

There is a common ratio of $\frac{1}{6}$. The sequence is geometric.

Step 2 Use the formula for a geometric sequence.

$a_n = r \cdot a_{n-1}$ $\quad$ Recursive formula for geometric sequence

$a_n = \frac{1}{6}a_{n-1}$ $\quad$ $r = \frac{1}{6}$

Step 3 The first term a_1 is 216 and $n \geq 2$.

A recursive formula for the sequence is $a_1 = 216$, $a_n = \frac{1}{6}a_{n-1}$, $n \geq 2$.

Exercises

Write a recursive formula for each sequence.

1. 22, 16, 10, 4, …

$a_1 = 22, a_n = a_{n-1} - 6, n \geq 2$

2. −8, −3, 2, 7, …

$a_1 = -8, a_n = a_{n-1} + 5, n \geq 2$

3. 5, 15, 45, 135, …

$a_1 = 5, a_n = 3a_{n-1}, n \geq 2$

4. 243, 81, 27, 9, …

$a_1 = 243, a_n = \frac{1}{3}a_{n-1}, n \geq 2$

5. −3, 14, 31, 48, …

$a_1 = -3, a_n = a_{n-1} + 17, n \geq 2$

6. 8, −20, 50, −125, …

$a_1 = 8, a_n = -2.5a_{n-1}, n \geq 2$

Lesson 7-8