8-1 Study Guide and Intervention

Adding and Subtracting Polynomials

Polynomials in Standard Form A **polynomial** is a monomial or a sum of monomials. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. Polynomials with more than three terms have no special name. The **degree** of a monomial is the sum of the exponents of all its variables. The **degree of the polynomial** is the same as the degree of the monomial term with the highest degree.

The terms of a polynomial are usually arranged so that the terms are in order from greatest degree to least degree. This is called the **standard form of a polynomial.**

> **Example** Determine whether each expression is a polynomial. If so, identify the polynomial as a *monomial*, *binomial*, or *trinomial*. Then find the degree of the polynomial.

Expression	Polynomial?	Monomial, Binomial, or Trinomial?	Degree of the Polynomial
$3x - 7xyz$	Yes. $3x - 7xyz = 3x + (-7xyz)$, which is the sum of two monomials	binomial	3
-25	Yes. -25 is a real number.	monomial	0
$7n^3 + 3n^{-4}$	No. $3n^{-4} = \dfrac{3}{n^4}$, which is not a monomial	none of these	—
$9x^3 + 4x + x + 4 + 2x$	Yes. The expression simplifies to $9x^3 + 7x + 4$, which is the sum of three monomials	trinomial	3

Exercises

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a *monomial*, *binomial*, or *trinomial*.

1. 36 **yes; 0; monomial**

2. $\dfrac{3}{q^2} + 5$ **no**

3. $7x - x + 5$ **yes; 1; binomial**

4. $8g^2h - 7gh + 2$ **yes; 3; trinomial**

5. $\dfrac{1}{4y^2} + 5y - 8$ **no**

6. $6x + x^2$ **yes; 2; binomial**

Write each polynomial in standard form. Identify the leading coefficient.

7. $x^3 + x^5 - x^2$
   $x^5 + x^3 - x^2$; 1

8. $x^4 + 4x^3 - 7x^5 + 1$
   $-7x^5 + x^4 + 4x^3 + 1$; -7

9. $-3x^6 - x^5 + 2x^8$
   $2x^8 - 3x^6 - x^5$; 2

10. $2x^7 - x^8$
    $-x^8 + 2x^7$; -1

11. $3x + 5x^4 - 2 - x^2$
    $5x^4 - x^2 + 3x - 2$; 5

12. $-2x^4 + x - 4x^5 + 3$
    $-4x^5 - 2x^4 + x + 3$; -4

8-1   Study Guide and Intervention   *(continued)*

Adding and Subtracting Polynomials

Add and Subtract Polynomials To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms vertically. **Like terms** are monomial terms that are either identical or differ only in their coefficients, such as $3p$ and $-5p$ or $2x^2y$ and $8x^2y$.

You can subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with by adding its additive inverse. To find the additive inverse of a polynomial, replace each term with its additive inverse or opposite.

Example Find $(3x^2 + 2x - 6) - (2x + x^2 + 3)$.

Horizontal Method

Use additive inverses to rewrite as addition. Then group like terms.

$(3x^2 + 2x - 6) - (2x + x^2 + 3)$
$= (3x^2 + 2x - 6) + [(-2x) + (-x^2) + (-3)]$
$= [3x^2 + (-x^2)] + [2x + (-2x)] + [-6 + (-3)]$
$= 2x^2 + (-9)$
$= 2x^2 - 9$

The difference is $2x^2 - 9$.

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{r} 3x^2 + 2x - 6 \\ (-)\ \ x^2 + 2x + 3 \end{array}$$

$$\begin{array}{r} 3x^2 + 2x - 6 \\ (+) -x^2 - 2x - 3 \\ \hline 2x^2 \qquad\ \ -9 \end{array}$$

The difference is $2x^2 - 9$.

Exercises

Find each sum or difference.

1. $(4a - 5) + (3a + 6)$
$7a + 1$

2. $(6x + 9) + (4x^2 - 7)$
$4x^2 + 6x + 2$

3. $(6xy + 2y + 6x) + (4xy - x)$
$10xy + 5x + 2y$

4. $(x^2 + y^2) + (-x^2 + y^2)$
$2y^2$

5. $(3p^2 - 2p + 3) + (p^2 - 7p + 7)$
$4p^2 - 9p + 10$

6. $(2x^2 + 5xy + 4y^2) + (-xy - 6x^2 + 2y^2)$
$-4x^2 + 4xy + 6y^2$

7. $(8p - 5r) - (-6p^2 + 6r - 3)$
$6p^2 + 8p - 11r + 3$

8. $(8x^2 - 4x - 3) - (-2x - x^2 + 5)$
$9x^2 - 2x - 8$

9. $(3x^2 - 2x) - (3x^2 + 5x - 1)$
$-7x + 1$

10. $(4x^2 + 6xy + 2y^2) - (-x^2 + 2xy - 5y^2)$
$5x^2 + 4xy + 7y^2$

11. $(2h - 6j - 2k) - (-7h - 5j - 4k)$
$9h - j + 2k$

12. $(9xy^2 + 5xy) - (-2xy - 8xy^2)$
$17xy^2 + 7xy$