

8-6 Study Guide and Intervention

Solving $x^2 + bx + c = 0$

Factor $x^2 + bx + c$ To factor a trinomial of the form $x^2 + bx + c$, find two integers, m and p , whose sum is equal to b and whose product is equal to c .

Factoring $x^2 + bx + c$	$x^2 + bx + c = (x + m)(x + p)$, where $m + p = b$ and $mp = c$.
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Example 1 Factor each polynomial.

a. $x^2 + 7x + 10$

In this trinomial, $b = 7$ and $c = 10$.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

Since $2 + 5 = 7$ and $2 \cdot 5 = 10$, let $m = 2$ and $p = 5$.

$$x^2 + 7x + 10 = (x + 5)(x + 2)$$

b. $x^2 - 8x + 7$

In this trinomial, $b = -8$ and $c = 7$.

Notice that $m + p$ is negative and mp is positive, so m and p are both negative. Since $-7 + (-1) = -8$ and $(-7)(-1) = 7$, $m = -7$ and $p = -1$.

$$x^2 - 8x + 7 = (x - 7)(x - 1)$$

Example 2 Factor $x^2 + 6x - 16$.

In this trinomial, $b = 6$ and $c = -16$. This means $m + p$ is positive and mp is negative. Make a list of the factors of -16 , where one factor of each pair is positive.

Factors of -16	Sum of Factors
1, -16	-15
-1 , 16	15
2, -8	-6
-2 , 8	6

Therefore, $m = -2$ and $p = 8$.

$$x^2 + 6x - 16 = (x - 2)(x + 8)$$

Exercises

Factor each polynomial.

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|---|--|---|
| 1. $x^2 + 4x + 3$
$(x + 3)(x + 1)$ | 2. $m^2 + 12m + 32$
$(m + 4)(m + 8)$ | 3. $r^2 - 3r + 2$
$(r - 2)(r - 1)$ |
| 4. $x^2 - x - 6$
$(x - 3)(x + 2)$ | 5. $x^2 - 4x - 21$
$(x - 7)(x + 3)$ | 6. $x^2 - 22x + 121$
$(x - 11)(x - 11)$ |
| 7. $t^2 - 4t - 12$
$(t + 2)(t - 6)$ | 8. $p^2 - 16p + 64$
$(p - 8)(p - 8)$ | 9. $9 - 10x + x^2$
$(9 - x)(1 - x)$ |
| 10. $x^2 + 6x + 5$
$(x + 5)(x + 1)$ | 11. $a^2 + 8a - 9$
$(a - 1)(a + 9)$ | 12. $y^2 - 7y - 8$
$(y - 8)(y + 1)$ |
| 13. $x^2 - 2x - 3$
$(x - 3)(x + 1)$ | 14. $y^2 + 14y + 13$
$(y + 1)(y + 13)$ | 15. $m^2 + 9m + 20$
$(m + 4)(m + 5)$ |
| 16. $x^2 + 12x + 20$
$(x + 10)(x + 2)$ | 17. $a^2 - 14a + 24$
$(a - 2)(a - 12)$ | 18. $18 + 11y + y^2$
$(9 + y)(2 + y)$ |
| 19. $x^2 + 2xy + y^2$
$(x + y)(x + y)$ | 20. $a^2 - 4ab + 4b^2$
$(a - 2b)(a - 2b)$ | 21. $x^2 + 6xy - 7y^2$
$(x + 7y)(x - y)$ |

8-6 Study Guide and Intervention *(continued)***Solving $x^2 + bx + c = 0$**

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve many equations of the form $x^2 + bx + c = 0$.

Example 1 Solve $x^2 + 6x = 7$. Check your solutions.

$x^2 + 6x = 7$	Original equation
$x^2 + 6x - 7 = 0$	Rewrite equation so that one side equals 0.
$(x - 1)(x + 7) = 0$	Factor.
$x - 1 = 0$ or $x + 7 = 0$	Zero Product Property
$x = 1$ $x = -7$	Solve each equation.

The solution set is $\{1, -7\}$. Since $1^2 + 6(1) = 7$ and $(-7)^2 + 6(-7) = 7$, the solutions check.

Example 2 **ROCKET LAUNCH** The formula $h = vt - 16t^2$ gives the height h of a rocket after t seconds when the initial velocity v is given in feet per second. If a rocket is fired with initial velocity 2288 feet per second, how many seconds will it take for the rocket to reach a height of 6720 feet?

$h = vt - 16t^2$	Formula
$6720 = 2288t - 16t^2$	Substitute.
$0 = -16t^2 + 2288t - 6720$	Rewrite equation so that one side equals 0.
$0 = -16(t - 143t + 420)$	Factor out GCF.
$0 = -16(t - 3)(t - 140)$	Factor
$t - 3 = 0$ or $t - 140 = 0$	Zero Product Property
$t = 3$ $t = 140$	Solve each equation.

The rocket reaches 6720 feet in 3 seconds and again in 140 seconds, or 2 minutes 20 seconds after launch.

Exercises

Solve each equation. Check the solutions.

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|--------------------------------------|--------------------------------------|--|
| 1. $x^2 - 4x + 3 = 0$ {1, 3} | 2. $y^2 - 5y + 4 = 0$ {1, 4} | 3. $m^2 + 10m + 9 = 0$ {-1, -9} |
| 4. $x^2 = x + 2$ {-1, 2} | 5. $x^2 - 4x = 5$ {-1, 5} | 6. $x^2 - 12x + 36 = 0$ {6} |
| 7. $t^2 - 8 = -7t$ {-8, 1} | 8. $p^2 = 9p - 14$ {2, 7} | 9. $-9 - 8x + x^2 = 0$ {-1, 9} |
| 10. $x^2 + 6 = 5x$ {2, 3} | 11. $a^2 = 11a - 18$ {2, 9} | 12. $y^2 - 8y + 15 = 0$ {3, 5} |
| 13. $x^2 = 24 - 10x$ {-12, 2} | 14. $a^2 - 18a = -72$ {6, 12} | 15. $b^2 = 10b - 16$ {2, 8} |

Use the formula $h = vt - 16t^2$ to solve each problem.

- 16. FOOTBALL** A punter can kick a football with an initial velocity of 48 feet per second. How many seconds will it take for the ball to first reach a height of 32 feet? **1 second**
- 17. ROCKET LAUNCH** If a rocket is launched with an initial velocity of 1600 feet per second, when will the rocket be 14,400 feet high? **at 10 seconds and at 90 seconds**