8-7 Study Guide and Intervention

Solving $ax^2 + bx + c = 0$

Factor $ax^2 + bx + c$ To factor a trinomial of the form $ax^2 + bx + c$, find two integers, m and p whose product is equal to ac and whose sum is equal to b. If there are no integers that satisfy these requirements, the polynomial is called a **prime polynomial**.

Example 1 **Factor $2x^2 + 15x + 18$.**

In this example, $a = 2$, $b = 15$, and $c = 18$. You need to find two numbers that have a sum of 15 and a product of $2 \cdot 18$ or 36. Make a list of the factors of 36 and look for the pair of factors with a sum of 15.

Factors of 36	Sum of Factors
1, 36	37
2, 18	20
3, 12	15

Use the pattern $ax^2 + mx + px + c$, with $a = 2$, $m = 3$, $p = 12$, and $c = 18$.

$$\begin{aligned} 2x^2 + 15x + 18 &= 2x^2 + 3x + 12x + 18 \\ &= (2x^2 + 3x) + (12x + 18) \\ &= x(2x + 3) + 6(2x + 3) \\ &= (x + 6)(2x + 3) \end{aligned}$$

Therefore, $2x^2 + 15x + 18 = (x + 6)(2x + 3)$.

Example 2 **Factor $3x^2 - 3x - 18$.**

Note that the GCF of the terms $3x^2$, $3x$, and 18 is 3. First factor out this GCF.

$3x^2 - 3x - 18 = 3(x^2 - x - 6)$.

Now factor $x^2 - x - 6$. Since $a = 1$, find the two factors of -6 with a sum of -1.

Factors of –6	Sum of Factors
1, –6	–5
–1, 6	5
–2, 3	1
2, –3	–1

Now use the pattern $(x + m)(x + p)$ with $m = 2$ and $p = -3$.

$x^2 - x - 6 = (x + 2)(x - 3)$

The complete factorization is

$3x^2 - 3x - 18 = 3(x + 2)(x - 3)$.

Exercises

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

1. $2x^2 - 3x - 2$
(2x + 1)(x – 2)

2. $3m^2 - 8m - 3$
(3*m* + 1)(*m* – 3)

3. $16r^2 - 8r + 1$
(4*r* – 1)(4*r* – 1)

4. $6x^2 + 5x - 6$
(2x + 3)(3x – 2)

5. $3x^2 + 2x - 8$
(3x – 4)(x + 2)

6. $18x^2 - 27x - 5$
(3x – 5)(6x + 1)

7. $2a^2 + 5a + 3$
(2*a* + 3)(*a* + 1)

8. $18y^2 + 9y - 5$
(6*y* + 5)(3*y* – 1)

9. $-4t^2 + 19t - 21$
(4*t* – 7)(3 – *t*)

10. $8x^2 - 4x - 24$
2(2x – 4)(2x + 3)

11. $28p^2 + 60p - 25$
(2*p* + 5)(14*p* – 5)

12. $48x^2 + 22x - 15$
(6x + 5)(8x – 3)

13. $3y^2 - 6y - 24$
3(*y* + 2)(*y* – 4)

14. $4x^2 + 26x - 48$
2(x + 8)(2x – 3)

15. $8m^2 - 44m + 48$
4(2*m* – 3)(*m* – 4)

16. $6x^2 - 7x + 18$
prime

17. $2a^2 - 14a + 18$
$2(a^2 - 7a + 9)$

18. $18 + 11y + 2y^2$
prime

Lesson 8-7

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8-7 Study Guide and Intervention *(continued)*

Solving $ax^2 + bx + c = 0$

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve some equations of the form $ax^2 + bx + c = 0$.

Example **Solve $12x^2 + 3x = 2 - 2x$. Check your solutions.**

$12x^2 + 3x = 2 - 2x$	Original equation
$12x^2 + 5x - 2 = 0$	Rewrite equation so that one side equals 0.
$(3x + 2)(4x - 1) = 0$	Factor the left side.
$3x + 2 = 0$ or $4x - 1 = 0$	Zero Product Property
$x = -\frac{2}{3}$ $\quad x = \frac{1}{4}$	Solve each equation.

The solution set is $\left\{-\frac{2}{3}, \frac{1}{4}\right\}$.

Since $12\left(-\frac{2}{3}\right)^2 + 3\left(-\frac{2}{3}\right) = 2 - 2\left(-\frac{2}{3}\right)$ and $12\left(\frac{1}{4}\right)^2 + 3\left(\frac{1}{4}\right) = 2 - 2\left(\frac{1}{4}\right)$, the solutions check.

Exercises

Solve each equation. Check the solutions.

1. $8x^2 + 2x - 3 = 0$ $\left\{\frac{1}{2}, -\frac{3}{4}\right\}$

2. $3n^2 - 2n - 5 = 0$ $\left\{-1, \frac{5}{3}\right\}$

3. $2d^2 - 13d - 7 = 0$ $\left\{-\frac{1}{2}, 7\right\}$

4. $4x^2 = x + 3$ $\left\{1, -\frac{3}{4}\right\}$

5. $3x^2 - 13x = 10$ $\left\{-\frac{2}{3}, 5\right\}$

6. $6x^2 - 11x - 10 = 0$ $\left\{-\frac{2}{3}, \frac{5}{2}\right\}$

7. $2k^2 - 40 = -11k$ $\left\{-8, \frac{5}{2}\right\}$

8. $2p^2 = -21p - 40$ $\left\{-\frac{5}{2}, -8\right\}$

9. $-7 - 18x + 9x^2 = 0$ $\left\{\frac{7}{3}, -\frac{1}{3}\right\}$

10. $12x^2 - 15 = -8x$ $\left\{-\frac{3}{2}, \frac{5}{6}\right\}$

11. $7a^2 = -65a - 18$ $\left\{-\frac{2}{7}, -9\right\}$

12. $16y^2 - 2y - 3 = 0$ $\left\{\frac{1}{2}, -\frac{3}{8}\right\}$

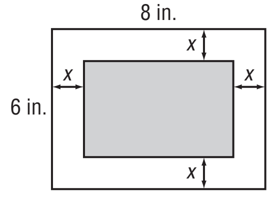
13. $8x^2 + 5x = 3 + 7x$ $\left\{\frac{3}{4}, -\frac{1}{2}\right\}$

14. $4a^2 - 18a + 5 = 15$ $\left\{-\frac{1}{2}, 5\right\}$

15. $3b^2 - 18b = 10b - 49$ $\left\{\frac{7}{3}, 7\right\}$

16. The difference of the squares of two consecutive odd integers is 24. Find the integers.
–5, –7 and 5, 7

17. GEOMETRY The length of a Charlotte, North Carolina, conservatory garden is 20 yards greater than its width. The area is 300 square yards. What are the dimensions?
30 yd by 10 yd



18. GEOMETRY A rectangle with an area of 24 square inches is formed by cutting strips of equal width from a rectangular piece of paper. Find the dimensions of the new rectangle if the original rectangle measures 8 inches by 6 inches. **6 in. by 4 in.**

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