

# 8-8 Study Guide and Intervention

## Differences of Squares

**Factor Differences of Squares** The binomial expression  $a^2 - b^2$  is called the **difference of two squares**. The following pattern shows how to factor the difference of squares.

Difference of Squares	$a^2 - b^2 = (a - b)(a + b) = (a + b)(a - b)$ .
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### Example 1 Factor each polynomial.

a.  $n^2 - 64$

$$\begin{aligned} n^2 - 64 \\ &= n^2 - 8^2 && \text{Write in the form } a^2 - b^2. \\ &= (n + 8)(n - 8) && \text{Factor.} \end{aligned}$$

b.  $4m^2 - 81n^2$

$$\begin{aligned} 4m^2 - 81n^2 \\ &= (2m)^2 - (9n)^2 && \text{Write in the form } a^2 - b^2. \\ &= (2m - 9n)(2m + 9n) && \text{Factor.} \end{aligned}$$

### Example 2 Factor each polynomial.

a.  $50a^2 - 72$

$$\begin{aligned} 50a^2 - 72 \\ &= 2(25a^2 - 36) && \text{Find the GCF.} \\ &= 2[(5a)^2 - 6^2] && 25a^2 = 5a \cdot 5a \text{ and } 36 = 6 \cdot 6 \\ &= 2(5a + 6)(5a - 6) && \text{Factor the difference of squares.} \end{aligned}$$

b.  $4x^4 + 8x^3 - 4x^2 - 8x$

$$\begin{aligned} 4x^4 + 8x^3 - 4x^2 - 8x &&& \text{Original polynomial} \\ &= 4x(x^3 + 2x^2 - x - 2) && \text{Find the GCF.} \\ &= 4x[(x^3 + 2x^2) - (x + 2)] && \text{Group terms.} \\ &= 4x[x^2(x + 2) - 1(x + 2)] && \text{Find the GCF.} \\ &= 4x[(x^2 - 1)(x + 2)] && \text{Factor by grouping.} \\ &= 4x[(x - 1)(x + 1)(x + 2)] && \text{Factor the difference of squares.} \end{aligned}$$

## Exercises

Factor each polynomial.

- |   |   |  |
|---|---|--|
| 1. $x^2 - 81$<br>$(x + 9)(x - 9)$             | 2. $m^2 - 100$<br>$(m + 10)(m - 10)$        | 3. $16n^2 - 25$<br>$(4n - 5)(4n + 5)$                      |
| 4. $36x^2 - 100y^2$<br>$(6x + 10y)(6x - 10y)$ | 5. $49x^2 - 36$<br>$(7x + 6)(7x - 6)$       | 6. $16a^2 - 9b^2$<br>$(4a - 3b)(4a + 3b)$                  |
| 7. $225b^2 - a^2$<br>$(15b - a)(15b + a)$     | 8. $72p^2 - 50$<br>$2(6p + 5)(6p - 5)$      | 9. $-2 + 2x^2$<br>$2(x - 1)(x + 1)$                        |
| 10. $-81 + a^4$<br>$(a - 3)(a + 3)(a^2 + 9)$  | 11. $6 - 54a^2$<br>$6(1 + 3a)(1 - 3a)$      | 12. $8y^2 - 200$<br>$8(y + 5)(y - 5)$                      |
| 13. $4x^3 - 100x$<br>$4x(x + 5)(x - 5)$       | 14. $2y^4 - 32y^2$<br>$2y^2(y + 4)(y - 4)$  | 15. $8m^3 - 128m$<br>$8m(m + 4)(m - 4)$                    |
| 16. $4x^2 - 25$<br>$(2x + 5)(2x - 5)$         | 17. $2a^3 - 98ab^2$<br>$2a(a - 7b)(a + 7b)$ | 18. $18y^2 - 72y^4$<br>$18y^2(1 - 2y)(1 + 2y)$             |
| 19. $169x^3 - x$<br>$x(13x + 1)(13x - 1)$     | 20. $3a^4 - 3a^2$<br>$3a^2(a + 1)(a - 1)$   | 21. $3x^4 + 6x^3 - 3x^2 - 6x$<br>$3x(x - 1)(x + 1)(x + 2)$ |

**8-8 Study Guide and Intervention** *(continued)***Differences of Squares**

**Solve Equations by Factoring** Factoring and the Zero Product Property can be used to solve equations that can be written as the product of any number of factors set equal to 0.

**Example** Solve each equation. Check your solutions.

a.  $x^2 - \frac{1}{25} = 0$

$$x^2 - \frac{1}{25} = 0 \quad \text{Original equation}$$

$$x^2 - \left(\frac{1}{5}\right)^2 = 0 \quad x^2 = x \cdot x \text{ and } \frac{1}{25} = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$$

$$\left(x + \frac{1}{5}\right)\left(x - \frac{1}{5}\right) = 0 \quad \text{Factor the difference of squares.}$$

$$x + \frac{1}{5} = 0 \quad \text{or} \quad x - \frac{1}{5} = 0 \quad \text{Zero Product Property}$$

$$x = -\frac{1}{5} \quad x = \frac{1}{5} \quad \text{Solve each equation.}$$

The solution set is  $\left\{-\frac{1}{5}, \frac{1}{5}\right\}$ . Since  $\left(-\frac{1}{5}\right)^2 - \frac{1}{25} = 0$  and  $\left(\frac{1}{5}\right)^2 - \frac{1}{25} = 0$ , the solutions check.

b.  $4x^3 = 9x$

$$4x^3 = 9x \quad \text{Original equation}$$

$$4x^3 - 9x = 0 \quad \text{Subtract } 9x \text{ from each side.}$$

$$x(4x^2 - 9) = 0 \quad \text{Factor out the GCF of } x.$$

$$x[(2x)^2 - 3^2] = 0 \quad 4x^2 = 2x \cdot 2x \text{ and } 9 = 3 \cdot 3$$

$$x[(2x)^2 - 3^2] = x[(2x - 3)(2x + 3)] \quad \text{Factor the difference of squares.}$$

$$x = 0 \quad \text{or} \quad (2x - 3) = 0 \quad \text{or} \quad (2x + 3) = 0 \quad \text{Zero Product Property}$$

$$x = 0 \quad x = \frac{3}{2} \quad x = -\frac{3}{2} \quad \text{Solve each equation.}$$

The solution set is  $\left\{0, \frac{3}{2}, -\frac{3}{2}\right\}$ .

Since  $4(0)^3 = 9(0)$ ,  $4\left(\frac{3}{2}\right)^3 = 9\left(\frac{3}{2}\right)$ , and  $4\left(-\frac{3}{2}\right)^3 = 9\left(-\frac{3}{2}\right)$ , the solutions check.

**Exercises**

Solve each equation by factoring. Check the solutions.

1.  $81x^2 = 49 \left\{\frac{7}{9}, -\frac{7}{9}\right\}$

2.  $36n^2 = 1 \left\{-\frac{1}{6}, \frac{1}{6}\right\}$

3.  $25d^2 - 100 = 0 \{2, -2\}$

4.  $\frac{1}{4}x^2 = 25 \{10, -10\}$

5.  $36 = \frac{1}{25}x^2 \{-30, 30\}$

6.  $\frac{49}{100} - x^2 = 0 \left\{-\frac{7}{10}, \frac{7}{10}\right\}$

7.  $9x^3 = 25x \left\{0, -\frac{5}{3}, \frac{5}{3}\right\}$

8.  $7a^3 = 175a \{0, -5, 5\}$

9.  $2m^3 = 32m \{0, -4, 4\}$

10.  $16y^3 = 25y \left\{0, -\frac{5}{4}, \frac{5}{4}\right\}$

11.  $\frac{1}{64}x^2 = 49 \{-56, 56\}$

12.  $4a^3 - 64a = 0 \{0, -4, 4\}$

13.  $3b^3 - 27b = 0 \{0, -3, 3\}$

14.  $\frac{9}{25}m^2 = 121 \left\{-\frac{55}{3}, \frac{55}{3}\right\}$

15.  $48n^3 = 147n \left\{0, -\frac{7}{4}, \frac{7}{4}\right\}$