NAME _____ DATE _____ PERIOD _____

9-4 Study Guide and Intervention

Solving Quadratic Equations by Completing the Square

Complete the Square Perfect square trinomials can be solved quickly by taking the square root of both sides of the equation. A quadratic equation that is not in perfect square form can be made into a perfect square by a method called **completing the square**.

Completing the Square

To complete the square for any quadratic equation of the form $x^2 + bx$:

Step 1 Find one-half of b, the coefficient of x.

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example **Find the value of c that makes $x^2 + 2x + c$ a perfect square trinomial.**

Step 1 Find $\frac{1}{2}$ of 2. $\frac{2}{2} = 1$

Step 2 Square the result of Step 1. $1^2 = 1$

Step 3 Add the result of Step 2 to $x^2 + 2x$. $x^2 + 2x + 1$

Thus, $c = 1$. Notice that $x^2 + 2x + 1$ equals $(x + 1)^2$.

Exercises

Find the value of c that makes each trinomial a perfect square.

1. $x^2 + 10x + c$ **25**

2. $x^2 + 14x + c$ **49**

3. $x^2 - 4x + c$ **4**

4. $x^2 - 8x + c$ **16**

5. $x^2 + 5x + c$ $\frac{25}{4}$

6. $x^2 + 9x + c$ $\frac{81}{4}$

7. $x^2 - 3x + c$ $\frac{9}{4}$

8. $x^2 - 15x + c$ $\frac{225}{4}$

9. $x^2 + 28x + c$ **196**

10. $x^2 + 22x + c$ **121**

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9-4 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Completing the Square

Solve by Completing the Square Since few quadratic expressions are perfect square trinomials, the method of **completing the square** can be used to solve some quadratic equations. Use the following steps to complete the square for a quadratic expression of the form $ax^2 + bx$.

Step 1	Find $\frac{b}{2}$.
Step 2	Find $\left(\frac{b}{2}\right)^2$.
Step 3	Add $\left(\frac{b}{2}\right)^2$ to $ax^2 + bx$.

Example **Solve $x^2 + 6x + 3 = 10$ by completing the square.**

$x^2 + 6x + 3 = 10$	Original equation
$x^2 + 6x + 3 - 3 = 10 - 3$	Subtract 3 from each side.
$x^2 + 6x = 7$	Simplify.
$x^2 + 6x + 9 = 7 + 9$	Since $\left(\frac{6}{2}\right)^2 = 9$, add 9 to each side.
$(x + 3)^2 = 16$	Factor $x^2 + 6x + 9$.
$x + 3 = \pm 4$	Take the square root of each side.
$x = -3 \pm 4$	Simplify.

$x = -3 + 4$ or $x = -3 - 4$

$= 1$ $\quad\quad$ $= -7$

The solution set is $\{-7, 1\}$.

Exercises

Solve each equation by completing the square. Round to the nearest tenth if necessary.

1. $x^2 - 4x + 3 = 0$
1, 3

2. $x^2 + 10x = -9$
−1, −9

3. $x^2 - 8x - 9 = 0$
−1, 9

4. $x^2 - 6x = 16$
−2, 8

5. $x^2 - 4x - 5 = 0$
−1, 5

6. $x^2 - 12x = 9$
−0.7, 12.7

7. $x^2 + 8x = 20$
−10, 2

8. $x^2 = 2x + 1$
−0.4, 2.4

9. $x^2 + 20x + 11 = -8$
−19, −1

10. $x^2 - 1 = 5x$
−0.2, 5.2

11. $x^2 = 22x + 23$
−1, 23

12. $x^2 - 8x = -7$
1, 7

13. $x^2 + 10x = 24$
−12, 2

14. $x^2 - 18x = 19$
−1, 19

15. $x^2 + 16x = -16$
−14.9, −1.1

16. $4x^2 = 24 + 4x$
−2, 3

17. $2x^2 + 4x + 2 = 8$
−3, 1

18. $4x^2 = 40x + 44$
−1, 11