9-6 Study Guide and Intervention

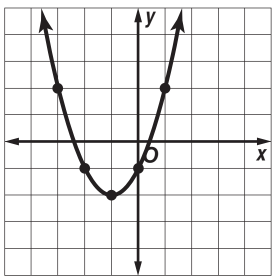
Analyzing Functions with Successive Differences

Identify Functions Linear functions, quadratic functions, and exponential functions can all be used to model data. The general forms of the equations are listed at the right.

Linear Function	$y = mx + b$
Quadratic Function	$y = ax^2 + bx + c$
Exponential Function	$y = ab^x$

You can also identify data as linear, quadratic, or exponential based on patterns of behavior of their y-values.

Example 1 **Graph the set of ordered pairs {(–3, 2), (–2, –1), (–1, –2), (0, –1), (1, 2)}. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.**



The ordered pairs appear to represent a quadratic function.

Example 2 **Look for a pattern in the table to determine which model best describes the data.**

x	–2	–1	0	1	2
y	4	2	1	0.5	0.25

Start by comparing the first differences.

$4 \xrightarrow[-2]{} 2 \xrightarrow[-1]{} 1 \xrightarrow[-0.5]{} 0.5 \xrightarrow[-0.25]{} 0.25$

The first differences are not all equal. The table does not represent a linear function. Find the second differences and compare.

$-2 \xrightarrow[+1]{} -1 \xrightarrow[+0.5]{} -0.5 \xrightarrow[+0.25]{} -0.25$

The table does not represent a quadratic function. Find the ratios of the y-values.

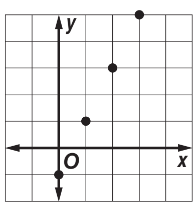
$4 \xrightarrow[\times 0.5]{} 2 \xrightarrow[\times 0.5]{} 1 \xrightarrow[\times 0.5]{} 0.5 \xrightarrow[\times 0.5]{} 0.25$

The ratios are equal. Therefore, the table can be modeled by an exponential function.

Exercises

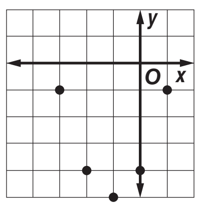
Graph each set of ordered pairs. Determine whether the ordered pairs represent a *linear* function, a *quadratic* function, or an *exponential* function.

1. (0, –1), (1, 1), (2, 3), (3, 5)



linear

2. (–3, –1), (–2, –4), (–1, –5), (0, –4), (1, –1)



quadratic

Look for a pattern in each table to determine which model best describes the data.

3.

x	–2	–1	0	1	2
y	6	5	4	3	2

linear

4.

x	–2	–1	0	1	2
y	6.25	2.5	1	0.4	0.16

exponential

9-6 Study Guide and Intervention *(continued)*

Analyzing Functions with Successive Differences

Write Equations Once you find the model that best describes the data, you can write an equation for the function.

Basic Forms	Linear Function	$y = mx + b$
	Quadratic Function	$y = ax^2$
	Exponential Function	$y = ab^x$

Example **Determine which model best describes the data. Then write an equation for the function that models the data.**

x	0	1	2	3	4
y	3	6	12	24	48

Step 1 Determine whether the data is modeled by a linear, quadratic, or exponential function.

First differences: $3 \xrightarrow[+3]{} 6 \xrightarrow[+6]{} 12 \xrightarrow[+12]{} 24 \xrightarrow[+24]{} 48$

Second differences: $3 \xrightarrow[+3]{} 6 \xrightarrow[+6]{} 12 \xrightarrow[+12]{} 24$

y-value ratios: $3 \xrightarrow[\times 2]{} 6 \xrightarrow[\times 2]{} 12 \xrightarrow[\times 2]{} 24 \xrightarrow[\times 2]{} 48$

The ratios of successive y-values are equal. Therefore, the table of values can be modeled by an exponential function.

Step 2 Write an equation for the function that models the data. The equation has the form $y = ab^x$. The y-value ratio is 2, so this is the value of the base.

$y = ab^x$ Equation for exponential function

$3 = a(2)^0$ $x = 0$, $y = 3$, and $b = 2$

$3 = a$ Simplify.

An equation that models the data is $y = 3 \cdot 2^x$. To check the results, you can verify that the other ordered pairs satisfy the function.

Exercises

Look for a pattern in each table of values to determine which model best describes the data. Then write an equation for the function that models the data.

1.

x	–2	–1	0	1	2
y	12	3	0	3	12

quadratic; $y = 3x^2$

2.

x	–1	0	1	2	3
y	–2	1	4	7	10

linear; $y = 3x + 1$

3.

x	–1	0	1	2	3
y	0.75	3	12	48	192

exponential; $y = 3 \cdot 4^x$