NAME ______________________________ DATE ____________ PERIOD ____________

9-7 Study Guide and Intervention

Special Functions

Lesson 9-7

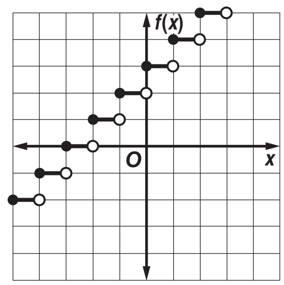
Step Functions The graph of a **step function** is a series of disjointed line segments. Because each part of a step function is linear, this type of function is called a **piecewise-linear function.**

One example of a step function is the greatest integer function, written as $f(x) = [\![x]\!]$, where $f(x)$ is the greatest integer not greater than x.

Example **Graph $f(x) = [\![x + 3]\!]$.**

Make a table of values using integer and noninteger values. On the graph, dots represent included points, and circles represent points that are excluded.

x	$x + 3$	$[\![x + 3]\!]$
–5	–2	–2
–3.5	–0.5	–1
–2	1	1
–0.5	2.5	2
1	4	4
2.5	5.5	5



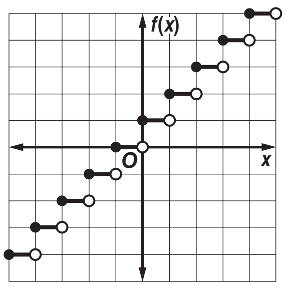
Because the dots and circles overlap, the domain is all real numbers. The range is all integers.

Exercises

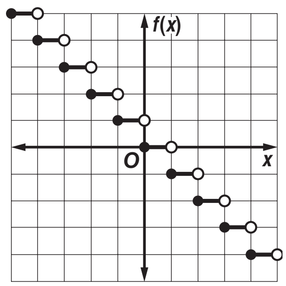
Graph each function. State the domain and range.

1–6. D = {all real numbers}; R = {all integers}

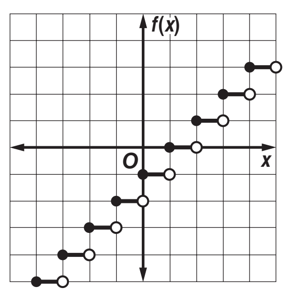
1. $f(x) = [\![x + 1]\!]$



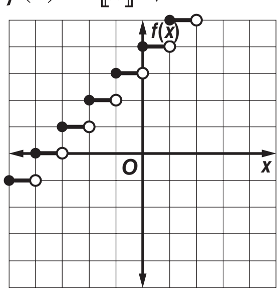
2. $f(x) = -[\![x]\!]$



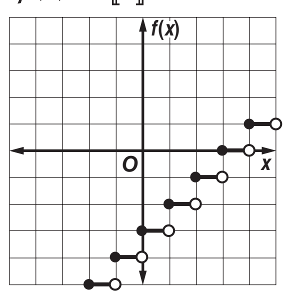
3. $f(x) = [\![x - 1]\!]$



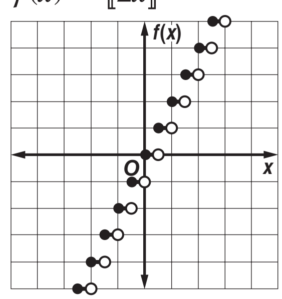
4. $f(x) = [\![x]\!] + 4$



5. $f(x) = [\![x]\!] - 3$



6. $f(x) = [\![2x]\!]$



9-7 Study Guide and Intervention *(continued)*

Special Functions

Absolute Value Functions Another type of piecewise-linear function is the **absolute value function**. Recall that the absolute value of a number is always nonnegative. So in the absolute value function, written as $f(x) = |x|$, all of the values of the range are nonnegative.

The absolute value function is called a **piecewise-defined function** because it can be written using two or more expressions.

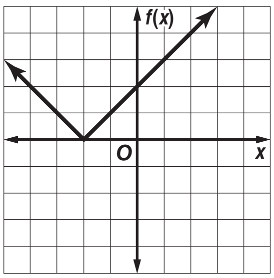
Example 1 **Graph $f(x) = |x + 2|$. State the domain and range.**

$f(x)$ cannot be negative, so the minimum point is $f(x) = 0$.

$f(x) = \|x + 2\|$	Original function
$0 = x + 2$	Replace $f(x)$ with 0.
$-2 = x$	Subtract 2 from each side.

Make a table. Include values for $x > -2$ and $x < -2$.

x	$f(x)$
–5	3
–4	2
–3	1
–2	0
–1	1
0	2
1	3
2	4

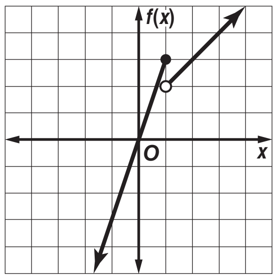


The domain is all real numbers. The range is all real numbers greater than or equal to 0.

Example 2 **Graph $f(x) = \begin{cases} x + 1 & \text{if } x > 1 \\ 3x & \text{if } x \le 1 \end{cases}$. State the domain and range.**

Graph the first expression. When $x > 1$, $f(x) = x + 1$. Since $x \neq 1$, place an open circle at (1, 2).

Next, graph the second expression. When $x \le 1$, $f(x) = 3x$. Since $x = 1$, place a closed circle at (1, 3).

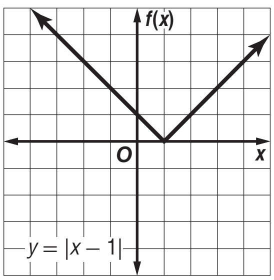


The domain and range are both all real numbers.

Exercises

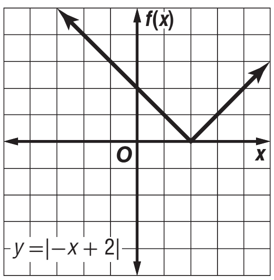
Graph each function. State the domain and range.

1. $f(x) = |x - 1|$



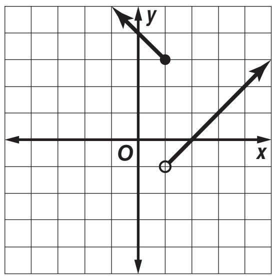
D = {all real numbers}; R = {y | $y > 0$}

2. $f(x) = |-x + 2|$



D = {all real numbers}; R = {y | $y > 0$}

3. $f(x) = \begin{cases} -x + 4 & \text{if } x \le 1 \\ x - 2 & \text{if } x > 1 \end{cases}$



D = {all real numbers}; R = {y | $y > -1$}